TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 68.

VORTICES AND THE RELATED PRINCIPLES OF HYDRODYNAMICS.

By

A. Betz, Göttingen.

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Not long ago the term "vortices" signified that we had reached the limit of our knowledge with reference to the phenomena of moving fluids. "Vortices were formed" was the expression and thereby the description of the process was simplified. Since Karman discovered behind obstructing bodies the system of vortices named for him and especially since well defined vortices have become the basis for accurate quantitative calculations, the conception of vortices has become a practical matter, although for many it is still a sort of magic word, incapable of actual demonstration. Aside from the difficulty of presentation inherent in all motions of fluids, this widespread confusion concerning vortices is partly due to the fact that the word vortex is not always used with the same meaning, so that misunderstandings easily arise. In this treatise, the conceptions concerning vortices will be illustrated by the simplest possible examples. Mathematical formulas and similar means of presentation, which, for the most part, do not help the understanding of persons not versed therein, have been avoided as much as possible. We have, instead, endeavored to demonstrate the phenomena by means of simple geometrical and me-

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chanical illustrations. For the sake of clearness, we will chiefly consider currents in only one plane, which can be represented readily by diagrams. In all planes parallel to the diagram the same process takes place. A special section will call attention briefly to some peculiarities of the phenomena in the case of flow in three dimensions.

We will first consider a few typical flows and obtain from them the conception of rotation. In the simplest form of flow, the fluid particles all move along parallel lines, as shown in Fig. 1, the length of the arrows being proportional to the velocity. If a small rod s is laid in such a current, it will retain its original angle with the current, no matter in what position it may have been placed.

Another simple flow is one in which the fluid revolves like a solid (Fig. 2). The velocity is then proportional to the distance from the center. If a rod s is placed in this current, it will also revolve about the center, its angular velocity remaining constant, so that, with every complete revolution about the center of the flow, it will also make a complete revolution about its inner extremity. Besides, in this case, the original location of the rod is immaterial, since its angular velocity is the same for all positions.

A third type of current is represented by Fig. 3, in which the stream lines are straight and parallel, as in Fig. 1, but the velocity is proportional to the distance from a line where the velocity is zero. If the rod is placed parallel to the flow (s_1), it will not turn. If it is placed across the flow (s_2), it turns in the direction indicated. Its angular velocity depends upon its direction and varies between zero in the position s_1 and a maximum value in the position s_2.

As the last example, we will again consider a flow with concentric circles, like case 2. In this case, however, the velocity is inversely proportional to the distance from the center (Fig. 4). A rod s_1 laid parallel to the current, will turn, like the hands of a clock, once during each revolution, while a radially placed stick (s_2) will turn in the opposite direction, on account of the speed distribution. As in the preceding case, the rotation speed depends on the position of the stick. Were we to determine the angular velocity of the stick for different positions, we would find their sum to be zero for every two positions at right angles to each other. Likewise we would be able to determine, in the third example, that the sum of the angular velocities for every two directions perpendicular to each other has a constant value. This value is designated as the "rotation" of the liquid. Currents 1 and 4 are accordingly free from rotation, while 3 and 4 rotate. The rotation in case 2 is twice as great as the angular velocity of the fluid revolving like a solid body.

A still clearer conception of this rotation can be obtained by imagining a square formed of four rods placed in the fluid (Figs. 1-4) and by considering how this square would be affected by the motion of the fluid. Fig. 5 shows the changes which would take place in the previously described typical motions. In the first case (uniform parallel motion), no change occurs, aside from a lateral displacement. In the second case, the figure turns without changing its shape. In cases 3 and 4, the square is changed into a rhombus. In case 4, the diagonals keep their original direction, one being lengthened while the other is shortened. From this exposition, it is evident that no rotation takes place. In case 3, we have the same deformation as in case 4, but accompanied by a rotation in which two of the sides return to their original direction. We will now consider whether and how by the application of external forces to a fluid originally at rest, we can set the latter in motion, so that the above-described currents will be produced. We will, however, refrain from taking an infinitely thin fluid, since it would require the application of infinitely great forces. It will suffice, if we generate currents like the four already described. We will thereby discover that we must apply two essentially different methods, according to the form of current to be generated, and we will obtain an important method of discrimination between different kinds of currents.

We can generate the first form (uniform parallel flow) by letting the fluid run out of a large tank A through a pipe (Fig. 6). For this purpose, we only need to see that the pressure in the pipe is less than in the tank. At some distance from the inflow, the streamlines in the pipe will be parallel. The pressure must then be constant throughout a cross-section of the pipe, for every variation in pressure in the cross-section would cause the fluid to deviate from its path and the streamlines would no longer be straight. Equality of pressure throughout a cross-section determines the uniformity of the velocity, since the latter is generated by the pressure difference between the pipe and the large tank A. It is, of course, assumed that no other force, aside from this pressure difference, exerts either an accelerating or retarding influence on the fluid. In particular, we must eliminate the influence of friction. We will see later that, in a very large number of important cases, this influence is so exceedingly small that it may be disregarded.

We must proceed in an entirely different manner in generating the second form of flow. It cannot be produced by a difference in pressure, since, in order to produce the greater velocity of the outer portions, care must be taken to have a lower pressure in these portions than in the inner slower portions. Such a pressure distribution is not possible, however, on account of the centrifugal forces of the fluid particles. These can continue in their circular paths, only when the centrifugal forces are offset by corresponding centripetal forces. With the circular paths, the outer pressure must therefore be greater than the inner. Such a pressure distribution is, however, just the opposite of that required for producing the desired speed distribution. This kind of a current can nevertheless be produced by rotating a liquid-filled cylinder about its axis. As a result of the viscosity (friction), the fluid gradually acquires a uniform rotary motion. The viscosity does not need to be especially strong, but the weaker it is, the longer it will take the liquid to acquire the desired motion.

Also the third form of flow cannot be generated by a difference in pressure, but only by the aid of viscosity. The straight streamlines are possible only where the pressure is uniform throughout the whole cross-section, perpendicular to the direction of the current. Such a pressure distribution would give, however, if no other influences were at work, a uniform velocity, as in the first example. With the aid of viscosity, the desired current can be generated by bringing the fluid between two smooth parallel walls, one of which is stationary, while the other is moving parallel to itself (Fig. 7). The fluid will then gradually acquire the desired motion.

In contrast with the last two examples, the fourth form of current (Fig. 4) can only be produced by a difference in pressure. As already mentioned in the second example, there is a necessary pressure increase toward the outside, due to the centrifugal force. If the velocity had to depend on the pressure alone, it would decrease with the pressure increase. As may be readily verified by calculation, the speed distribution in our example (inversely proportional to the radius) is such that the pressure (corresponding to the speed) exactly offsets the centrifugal force. The actual production of this form of current may take place in some such manner as indicated in Fig. 8. The fluid runs out of the large tank A through a pipe with a semicircular bend. As a result of the centrifugal force, the pressure will be greater and the velocity correspondingly smaller toward the outer than toward the inner side of this bend. On account of its nearness to the straight part of the pipe, there are some deviations from the desired form of current. The amount of this deviation varies directly as the ratio of the pipe diameter to the radius of the bend.

From these considerations it is evident that, in the absence of viscosity or when it is very small in comparison with the pressure differences, only certain forms of current can be produced, while other forms are mainly dependent on the effects of viscosity.

This distinction is of great practical importance, since the viscosity of air and water, which are the most important fluids from a technical standpoint, is so small that it is very often negligible. In all these cases, the possible forms of current are very limited and possess certain characteristics which greatly simplify the numerical treatment.

By comparing the classification of the current forms according to their origin, with the classification made on the basis of the motions of the small rod, we come to the noteworthy conclusion that both examples of non-rotary motion (types 1 and 4) are produced by pressure, while both examples of rotary motion are produced by friction. This distinction does not exist simply by chance in the examples chosen, but is grounded on facts and is of universal application.

The peculiar relation between the radius of bend of the stream at any point and the speed distribution along the radius, which was found in the fourth example to be the necessary consequence of the absence of friction, exists in every current generated by pressure with the elimination of friction influences. The centrifugal forces are determined by the radius of bend and the velocity. Therewith both the pressure increase and also (on account of the absence of friction) the corresponding decrease in velocity are entirely toward the outside.

Hence the important law: When a fluid current is generated from a condition of rest, it is free from rotation at all points where the fluid particles have not been influenced by friction.

We may accordingly expect any considerable rotation in a fluid, only when the forces produced by friction are of the same general order of magnitude as those generated by pressure. This occurs in the practically important cases, that is, in fluids of very slight viscosity (air and water), only when the velocity varies greatly within a relatively thin layer. By far the most important instance of this occurs in a flow along a stationary wall (Fig. 9). The fluid particles in immediate contact with the wall have a velocity of zero. With increasing distance from the wall, the velocity increases up to the value it would have without the influence of the wall. The longer the fluid runs along the wall, the thicker will be this boundary layer within which the velocity increases, since there is a constantly increasing number of particles retarded by friction. Within this boundary layer, there is a rotational motion which is stronger in proportion to the thinness of the layer, due to the more rapid speed acceleration. It is accordingly evident that rotational motion must be expected in all fluid particles flowing near a wall, and indeed, either in a thin layer with a strong rotational motion, or in a thicker layer with a feeble rotational motion. Such fluid particles may even pass from the wall to the interior of the fluid, where they retain their once acquired rotation a long time, since there is usually very little friction to increase or diminish it. (In a relatively long time the slight viscosity, however, finally eliminates the rotation.) These rotational regions usually form but a small part of the whole fluid. The simple laws for irrotational fluid motions may therefore be applied to the processes by making proper allowances for these small limited regions.

After these general remarks on rotational and irrotational motions and their formation in ordinary currents, let us again turn our attention to the fourth typical form of flow (Fig. 4), which has a circular but irrotational motion. If the rod, which served us for testing the rotation, is laid with its center over the center of the circle, it will then show the same rotation for every position and the shorter the rod employed, the quicker its rotation, since the velocity of the fluid increases up to the cen9 ter. While freedom from rotation has been established for every other part of the current, we find a rotational motion at this one point and indeed an infinitely swift one. In investigating the scientific significance of this phenomenon, we must bear in mind that a flow of this type is only possible when the center and a small surrounding tract is excluded, for it is impossible to generate infinitely great speeds. This fourth type is nevertheless an exceedingly important one in connection with which it is necessary to remember that this type of flow only exists beyond a certain radius, within which however there is another form of flow.

We can imagine the following example (Fig. 10). The fluid flows in concentric circles. Outside a certain circle, there is an irrotational flow of type 4 (Fig. 4), while within this circle (dotted area) there is a rotational motion of type 2 (Fig. 2). Such a combination is termed a vortex. (Any flow not free from rotational motions is often called a vortex. Since however it is contrary to ordinary usage to speak of the third type of flow (Fig. 3) as a vortex, we will apply the term only to types of flow having a well-defined nucleus about which the current flows.) We will call the region of rotational fluid the "nucleus" and the outer irrotational portion the "field" of the vortex. It is however not necessary for the rotational motion in the nucleus to be uniform, as in the chosen example. It is only necessary to have a limited region of rotational fluid particles surrounded by an irrotational flow, or at least by a current with less rotation.

Very different distributions of the rotational motion in the nucleus are possible for any given vortex field. Thus, for example, as shown in Fig. 11, the rotational fluid (dotted area) may be concentrated in a ring, outside of which there is no rotational motion and inside of which there is no motion at all. Since, as already stated, the rotational portion is generally very limited in the ordinary fluids of small viscosity, the nucleus is consequently very small and the phenomena which interest us play a far more important role in the vortex field. It is therefore of no consequence in most cases as to how the nucleus is constituted, provided it fits the vortex field. Since the nucleus is a matter of indifference, we may, for the sake of simplicity, assume it to have the form of a point and thus return to the fourth type of flow (Fig. 4). We must however bear in mind that this is only in the sense that the actual flow, outside of a certain small area, agrees with the current represented by Fig. 4, while within said area there are other phenomena, which however do not interest us in detail.

In order to describe the whole process in all its details, we need only to know, aside from the fact that it is free from rotation, the center and a single number, which, for instance, designates the velocity at a unit distance from the center. Instead of this last number, one 2π times as large is usually given, which is termed the "circulation." It is the product of the circumference of any selected streamline circle and the velocity on this circumference. This product is the same for every circle outside the nucleus, since the velocity decreases in the same ratio as the circumference increases. This "circulation" number defines the magnitude of the nucleus. If the rotation in the nucleus is uniform (like a solid body, type 2), then the "circulation" is equal to the product of the rotation times the area of the nucleus, for, if r represents the radius of the nucleus and ω the angular velocity (2ω the rotation), then the circumferential velocity of the nucleus is $r\omega$, the circumference is $2\pi r$, and the "circulation" is their product: $2\omega\pi r^2$, which equals the product of the area of the nucleus πr^2 times the rotation 2ω. If the rotation is not uniformly distributed, its mean value must be taken, which, multiplied by the area of the nucleus, will then give the "circulation" about the nucleus.

It seems strange at first that all the numerous fluid motions can be definitely determined from so few data (the location of the center and the value of the "circulation"). The explanation lies in the assumption that the flow is free from rotation with the exception of the small area of the nucleus. This assumption excludes all of the infinitely many possible varieties of flow, excepting this one alone, for all the others are rotational. (To be exact, the condition must be added that the velocity at a great distance from the nucleus must be nearly zero. This is of no consequence, however, for practical purposes.) Herein lies one of the very great simplifications which becomes possible just as soon as it is known that a flow is free from rotation. As we have already shown, this condition is often fulfilled by the principal part of the fluid.

We have first considered only the irrotational motion of type 4 and its relation to the nucleus. We can generalize the consideration and render it applicable to other irrotational flows, by combining two or more types of flow. By this is meant the following: If we have a flow with a simple vortex field and the center A_1 (Fig. 12), then there is a definite velocity v_1 at a definite point P. If we take another vortex field with its center A_2, then the velocity is v_2 at the point P. If we now combine the velocities v_1 and v_2, just as forces are combined in a parallelogram of forces, we obtain the resultant velocity v. By carrying out this process for every point of the space, we obtain a new velocity for every point. By means of these new velocities we obtain a new field of flow, and we say that it is done by combining or adding the two original fields. It may be demonstrated now that the rotation, produced by the flow resulting from the composition of the two flows, is equal to the sum of the rotations possessed by the two original component flows. Accordingly, if irrotational flows are combined, an irrotational flow results and, if a rotational field is combined with an irrotational field, the rotation remains unchanged. This law may be readily comprehended, if it is made clear that the velocities, which caused the motions of our testing rod, are added by the composition. Consequently, the angular speeds and rotations are also added. By such compositions we can now construct the various non-rotatory types of flow, for whose accurate definition it is sufficient to give the location of the vortex nucleus and its "circulation." If, therefore, we can follow the production and motion of the rotating fluid particles, so as to determine their distribution and the amount of their rotation, the course of all the remaining irrotational flow is readily found. It is not necessary for the rotating fluid particles to be separated into individual nuclei. In combining, we may even have the nuclei close to one another (Fig. 13) and thus finally obtain a vortex layer, as, for instance, in the boundary layer of a flow along a stationary wall. The transverse velocities between two nuclei are mutually reduced more completely, the less non-rotating fluid there is between them and disappears altogether when they are so close that along the vortex layer the rotation no longer exhibits any fluctuation (when, in particular, no irrotational parts are shoved between the rotating portions). Then there remains only the velocities along the vortex layer, hence a flow of the third type (Fig. 3). (It is for this reason that all rotational motions of a fluid are often called vortices.)

We have seen that it is only necessary to know the location of the rotating parts of the liquid and the amount of their rotation, in order to determine all the rest of the data. Many wonder at this fact and find it incomprehensible that anything so small as the nucleus of a vortex can exert a determining influence on all the rest of the current. They wrongly regard the nucleus as the mechanical cause of the vortex field. In reality, the irrotational flow is produced by pressure and where there is considerable friction, regions of rotational motion are generated, which are variously distributed by the motion of the main current. It may therefore be said much more truthfully that the whole flow generates and distributes the vortex nuclei. In any event, the size and distribution of these nuclei is very closely connected with the motion of the rest of the fluid, so that we may calculate backwards from the distribution of the nuclei to the corresponding flow, as one can determine a cause from its effect.

On account of the small influence exerted by the viscosity, a vortex, once generated, persists for a long time. In the presence of several nuclei, their relative position changes, but the fields belonging to the individual nuclei remain the same, only their form of combination varying. In the course of a long time, however, the viscosity takes effect and changes the individual nuclei. The neighboring parts of the fluid are likewise gradually set in rotation and themselves form components of the nucleus. Hence the nuclei gradually spread out. The "circulation" about the nucleus is not changed thereby, because we can always measure it at any desired distance from the nucleus, hence at so great a distance that the influence of the viscosity is not felt. If, however, the nucleus enlarges without increasing the "circulation," then the mean rotation of the fluid in the nucleus must have been reduced by being distributed over the larger area. In Fig. 14 the velocity distribution in a vortex is shown in two different phases, the longer arrows giving the original, and the shorter ones the subsequent velocities. In the part of the fluid which is free from rotation, the velocities remain unchanged, but they are smaller in the nucleus. The law that the "circulation" remains constant in this extension of the nucleus, holds good only so long as several nuclei do not merge into one another. When two nuclei rotating in opposite directions, merge, then their rotary motion is partially or wholly eliminated. If the circulations about these nuclei were originally equal and opposite, both eddies completely disappear after awhile. A similar phenomenon takes place when the fluid is confined by walls, the circulation being reduced as soon as the nuclei reach the stationary boundary, or rather the boundary layer, which rotates in the opposite direction to the nucleus. Such conditions are always present. In the vicinity of a vortex, either there are stationary walls (which generally participate in producing the vortex), or else, when the fluid is greatly extended, there are several vortices rotating in opposite directions, so that their sum is zero. In an infinitely extended fluid, the kinetic energy of a single vortex would be infinitely large. It is therefore impossible to generate such a vortex, but there must always be many rotating in contrary directions, whose energy is finite, if the sum of their "circulations" is zero. Consequently, all vortices gradually disappear from a fluid which is not acted upon for a long time by outside forces.

As already mentioned, our considerations have been confined to horizontal currents, since they are much easier to understand and still exhibit the essential phenomena. We must however call attention to a few special characteristics resulting from the great variety of natural phenomena. Since the flow is horizontal, the vortex nuclei must constitute vertical cylinders. Either they spread out to infinity in an infinitely extended fluid, or, if the fluid is confined between parallel walls, they end at these walls. In the general case of motion in space, the vortex nuclei are not straight lines, but form some sort of a curved tube around which the main current flows. The field of such a nucleus may likewise be definitely determined, if the location of the nucleus and the "circulation" about it are known. The calculation of the velocities is not quite so easy as for uniplanar flow, but is nevertheless comparatively simple. The same law holds good for the velocities of a vortex nucleus, as for the magnetic field of an electric conductor carrying a current. The velocity corresponds to the intensity of the magnetic field, the vortex nucleus to the conductor and the "circulation" to the strength of the electric current. Just as the intensity of the electric current is the same in every cross-section of the conductor, the "circulation" about a vortex nucleus cannot change throughout its extent. The vortex nucleus cannot end therefore at any point whatsoever in the fluid. It must either assume the form of a closed ring, or spread out to infinity, or end at the edges of the fluid. There is also the possibility that a vortex nucleus may split into several parts in which event the sum of the "circulations" of the individual parts remains the same, just like the current in a divided electric circuit. Such a nucleus may be conceived as a group of nuclei so close together that they appear like a single nucleus. The important law of constant value of the "circulation" follows from purely geometric relations, which are however not easily explained. It may be demonstrated that, when the "circulations" at two neighboring cross-sections of a nucleus differ, a rotation of the fluid, of the type 3 form, is produced by the resulting difference between the velocities v_1 and v_2. This is contrary however to the assumption that the flow outside the nucleus is free from rotation. A well-known example of a nonrectilinear nucleus is a nucleus ring, as illustrated by a smoke ring.

In the foregoing it has been shown how the conception of rotation and vortices is formed, what use this conception is in the consideration of fluid motion and what their most prominent characteristics are. In conclusion, I will give a few examples of the most common forms of vortices. When a current flows along a stationary wall, there is formed in the neighborhood of this wall a layer of rotating fluid (Fig. 16, left). If the wall breaks suddenly off at an angle, the rotating layer continues into the free fluid by reason of its momentum. It there forms the transition between the quiet and the flowing fluid (Fig. 16). Such vortex layers in a free fluid are not generally stable (excepting in very viscous fluids or at very low velocities). They have a tendency to develop into waves. These waves become constantly higher and finally assume spiral forms. Thereby the rotating liquid piles up against the center of the spirals and after awhile form, for the most part, individual nuclei at regular intervals (Fig. 16, right).

A sharp corner is not absolutely necessary for separating the rotational boundary layer from the stationary wall. It is only necessary, in the current along the wall, to have a sufficient increase of pressure in the fluid. This retards the boundary layer, reverses its slowest portions and thereby deflects the newly arriving masses of rotating fluid. Generally the formation of the individual vortices proceeds somewhat more rapidly than represented in Fig. 16, so that the intermediate steps often do not appear at all. This depends on the viscosity, the velocity and the thickness of the boundary layer. An interesting example of this kind of vortex formation is the production of the "Kármán" vortices which, under certain conditions, are formed behind an obstructing body. Fig. 17 represents a cylinder with such vortices. Vortices are formed alternately on either side, which, after reaching a certain magnitude are carried along by the current and then exhibit the regular staggered arrangement shown in the figure.

Another very important form of vortex is that produced behind airplane wings (or propeller blades). In order that lift may be exerted against the wing, the upward pressure must be greater than the downward pressure. The air has the tendency to equalize this pressure difference by flowing around the lateral edges of the wing (Fig. 18 above, aerofoil seen from behind). The boundary layer, resulting from this lateral flowing of the air on the upper and lower sides of the wing, remains behind in the space traversed by the wing. The rotating air is mainly behind the ends of the wing, since the highest lateral velocities are there. There are formed therefore two principal vortex streams stretching backward from the tips of the wing. The study of the current field belonging to these vortex streams has led to very fruitful results in the aerofoil theory. In order to avoid misunderstandings, let it be emphasized here that, aside from these main vortices, which are due to the lift and whose axes are essentially parallel to the direction of flight, there are also the vortices caused by the drag of the wing section, which, like the Karman eddies in Fig. 17, rotate alternately right and left, whereby their axes are practically parallel to the trailing edge of the wing and hence perpendicular to the flight direction. These are generally considerably weaker than the lift eddies.

As the rotational boundary layer, after its separation from the wall which produced it, separates into individual eddies, so it can under certain conditions, while still clinging to the wall, become unstable and form vortices. The current thus formed is represented by Fig. 19. Whether the boundary layer breaks up into such individual vortices (becoming turbulent) or not (laminated like Fig. 9), depends again on the viscosity, the velocity and the thickness of the boundary layer. Slight viscosity, great velocity and a considerable thickness of the boundary layer are necessary for the formation of the vortical (turbulent) boundary layer. (In comparison with the body of fluid on the edge of which the boundary layer is formed, this boundary layer is nevertheless very thin.) If these assumptions are to a great degree fulfilled, there is then produced not a single row of vortices, as in Fig. 19, but an irregular confusion of currents which can be followed only with difficulty. Similar irregular vortices may occur in place of the Karman vortices behind obstacles. To such phenomena applies the remark made at the beginning of this article, according to which the term "vortex" is used for motion phenomena that are not understood. In fact, these vortex-like currents are very common. In a very large proportion of fluid motions, we have to do with regularly formed vortices, and in these cases the term "vortex" is not the expression of our ignorance, but on the contrary, a conception which very greatly facilitates our numerical investigation of the phenomena of fluid currents.

Summary.

Starting with a few typical illustrations, the conception of rotation in a uniplanar fluid flow has been explained and it has been shown that no rotational motion can be produced by pressure

without friction. In general, it is only in proximity to stationary walls that there is any appreciable friction in comparison with the pressure, so that the vortex nuclei, which consist of rotating fluid portions, are mostly of very limited extent in comparison with the irrotational vortex field. The motion in the vortex field can be determined from a very few data concerning the location and strength ("circulation") of the vortex nucleus. In the course of time, the vortices are dissipated by the viscosity. In the motions of fluids in space, there is an analogy between the vortex field and the magnetic field of an electric conductor. The law, that the "circulation" throughout the vortex is constant, is explained. In conclusion, a few examples are cited of the most common forms of vortices.

Translated by the National Advisory Committee for Aeronautics.

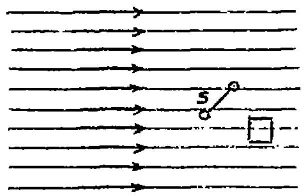

Fig. 1

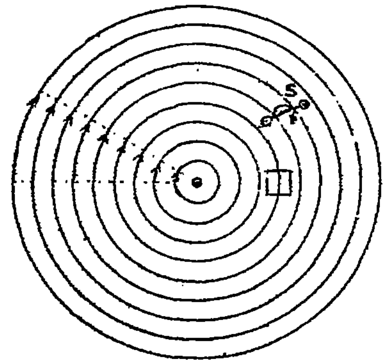

Fig. 2.

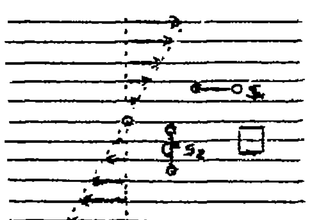

Fig. 3.

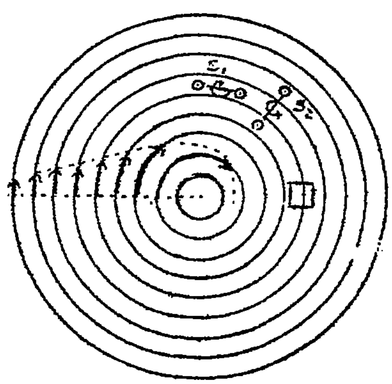

Fig. 4.

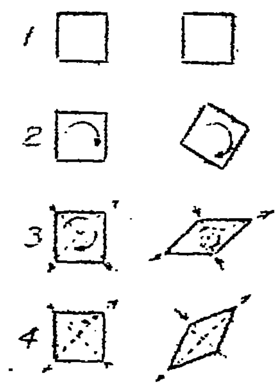

Fig. 5. Changes in shape of the squares shown in Figs. 1 to 4, resulting from flow of fluid.

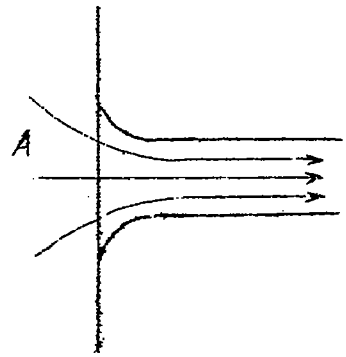

Fig. 6

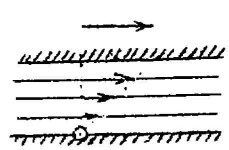

Fig. 7.

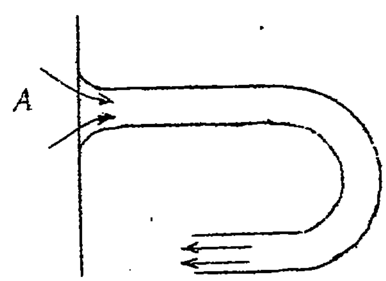

Fig. 3.

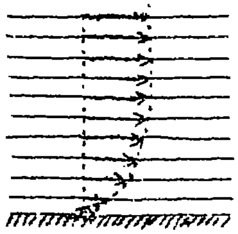

Fig. 9. Velocity Distribution Near a stationary wall.

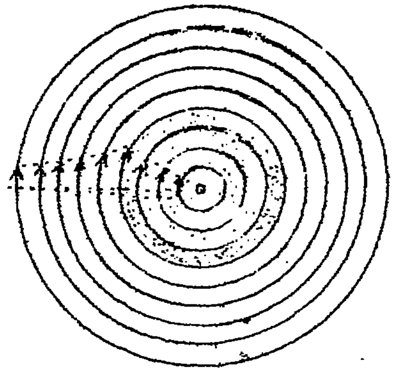

Fig. 10. Nucleus and field of a vortex.

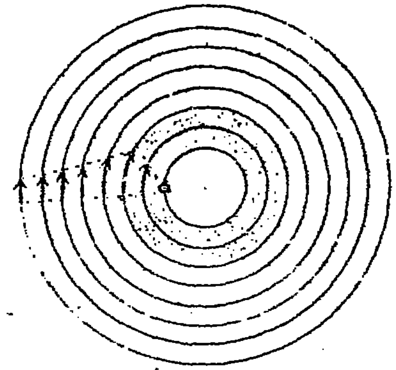

Fig. 11. Vortex with ring-shaped nucleus.

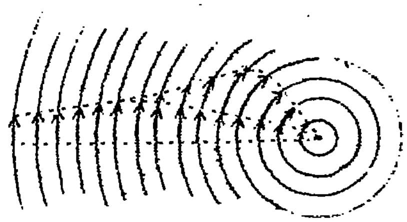

Fig. 14. Widening of the vortex nucleus.

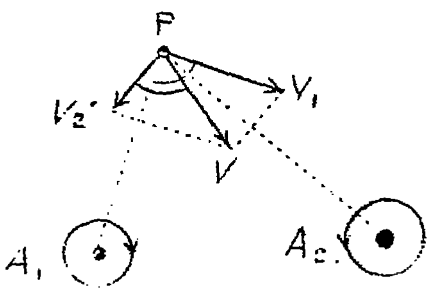

Fig. 12. Composition of 2 vortex fields.

Fig. 13.

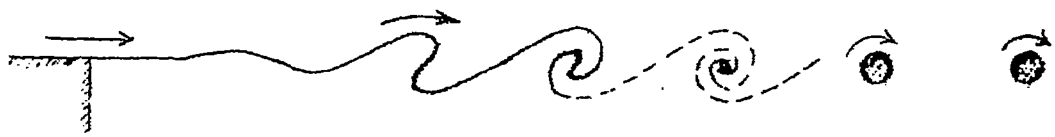

Fig. 16. Gradual formation of vortices from a boundary layer pushed into the free fluid.

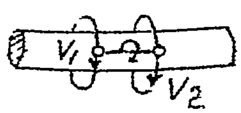

Fig. 15.

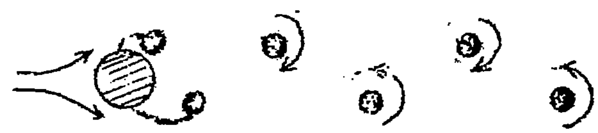

Fig. 17. Karman vortices behind a cylinder.

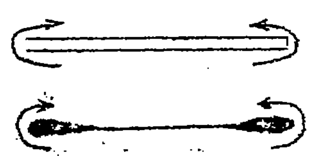

Fig. 18 Above: Flow around the lateral edges of an aerofoil (seen from behind.)
Below: Continuation of this motion, behind the aerofoil, around the rotary layer produced on the aerofoil.

Fig. 19. Boundary layer changed into individual vortices.